

Propositional and Predicate Logic - Second Homework

The deadline for this homework is November 23, 2023, at 14:00 (before the tutorials). You can submit your solution to my email Martin.Pilat@mff.cuni.cz (a scan on your handwriting is OK), or on paper before the tutorials.

You can get up to 5 points for the homework that count towards the credit requirements.

Let φ, ψ are two propositions over the set of variables $\mathbb{P} = \{p, q, r, s, t\}$:

$$\varphi : (p \vee r) \wedge (\neg p \vee q) \wedge (\neg q \vee \neg r), \quad (1)$$

$$\psi : (\neg s \vee \neg r) \wedge (\neg t \vee s) \wedge \neg p. \quad (2)$$

1. Using implication graph, decide if the proposition $\varphi \wedge \psi$ is satisfiable. If it is, find a satisfying assignment. (1 point)
2. Transform φ into CNF and DNF. (1 point)
3. Count the number of non-equivalent propositions θ over $\mathbb{P}' = \{p, q, r\}$ such that $\varphi \models \theta$. (1 point)
4. Let variables r, s, t represent (in this order) that "Rachel / Sarah / Tom is at school" and let $\mathbb{P} = \{r, s, t\}$. We know that
 - (i) If Tom is not at school, Sarah is also not there.
 - (ii) Rachel does not go to school without Sarah.
 - (iii) If Rachel is not at school, Tom is there.

We want to use the tableau method to show that these facts imply that Tom is at school. Therefore:

- (a) Write propositions $\varphi_1, \varphi_2, \varphi_3$ over \mathbb{P} expressing the facts (i)-(iii). (1 point)
- (b) Use the tableau method to show that T implies that Tom is at school, i.e. $T \models t$. (1 point)