

Propositional and Predicate Logic – Second Homework

The deadline for the homework is January 3rd, 2019 at 2:00 p.m. You can either hand me the solution on paper (preferably before the seminar on January 3rd), or send it via e-mail to Martin.Pilat@mff.cuni.cz (a scan/photo of your handwriting is OK).

The points obtained for this homework count towards the points required to obtain the credit for the seminar. Each question is worth 0.5 points, so you can get at most 2 points in total.

1. Let $\mathcal{A} = \langle \mathbb{Q}, +, \cdot \rangle$ be a structure in language $L = \langle +, \cdot \rangle$ with equality, where \mathbb{Q} is the set of rational numbers, “+” is the addition of rational numbers and “ \cdot ” is the multiplication of rational numbers.
 - (a) Find the substructure of \mathcal{A} generated by the set $\{1\}$, i.e. $\mathcal{A}\langle 1 \rangle$.
 - (b) Are there any other substructures of \mathcal{A} ?
 - (c) Are all the substructures of \mathcal{A} elementarily equivalent?
2. Let $F(x, y)$ represent that “there is a flight from x to y ” and let $C(x, y)$ represent that “there is a connection from x to y ”. Assume that
 - (a) From Prague you can fly to Bratislava, London and New York, and from New York to Paris,
 - (b) $(\forall x)(\forall y)(F(x, y) \rightarrow F(y, x))$,
 - (c) $(\forall x)(\forall y)(F(x, y) \rightarrow C(x, y))$,
 - (d) $(\forall x)(\forall y)(\forall z)((C(x, y) \wedge F(y, z)) \rightarrow C(x, z))$.

Prove by tableau method that there is a connection from Bratislava to Paris.

3. Convert the following formula into the prenex normal form and find its Skolem variant:

$$(\forall y)((\exists x)P(x, y) \rightarrow Q(y, z)) \wedge (\exists y)((\forall x)R(x, y) \vee Q(x, y)).$$

4. Let $T = \{(\exists y_1)(\exists y_2)(\forall x)(x = y_1 \vee x = y_2), \neg(\exists x)(f(x) = x)\}$ be a theory in language $L = \langle f \rangle$ with equality, where f is a unary function symbol.
 - (a) Is T a conservative extension of the theory $T_0 = \{(\exists y_1)(\exists y_2)(\forall x)(x = y_1 \vee x = y_2)\}$ in language $L' = \langle \rangle$ with equality?
 - (b) Are theories T and T_0 complete? Give an explanation.
 - (c) Is T equivalent to some open theory? Give an explanation.