

Propositional and Predicate Logic – Second Homework

The deadline for this homework is January 4, 2024 at 14:00. You can either hand me the solution on paper before the tutorials on Jan. 4, or send it via e-mail to Martin.Pilat@mff.cuni.cz (a scan/photo of your handwriting is OK).

The points obtained for this homework count towards the points required to obtain the credit for the seminar.

1. Let $\mathcal{A} = \langle \mathbb{Q}, +, \cdot \rangle$ be a structure in language $L = \langle +, \cdot \rangle$ with equality, where \mathbb{Q} is the set of rational numbers, “+” is the addition of rational numbers and “ \cdot ” is the multiplication of rational numbers.
 - (a) Find the substructure of \mathcal{A} generated by the set $\{1\}$, i.e. $\mathcal{A}\langle 1 \rangle$. (0.5 point)
 - (b) Are there any other substructures of \mathcal{A} ? (0.5 point)
 - (c) Are all the substructures of \mathcal{A} elementarily equivalent? (1 point)
2. Let $F(x, y)$ represent that “there is a flight from x to y ” and let $C(x, y)$ represent that “there is a connection from x to y ”. Assume that
 - (a) From Prague you can fly to Bratislava, London and New York, and from New York to Paris,
 - (b) $(\forall x)(\forall y)(F(x, y) \rightarrow F(y, x))$,
 - (c) $(\forall x)(\forall y)(F(x, y) \rightarrow C(x, y))$,
 - (d) $(\forall x)(\forall y)(\forall z)((C(x, y) \wedge F(y, z)) \rightarrow C(x, z))$.Prove by tableau method that there is a connection from Bratislava to Paris. (1 point)
3. Let $T = \{(\exists y_1)(\exists y_2)(\forall x)(x = y_1 \vee x = y_2), \neg(\exists x)(f(x) = x)\}$ be a theory in language $L = \langle f \rangle$ with equality, where f is a unary function symbol.
 - (a) Is T a conservative extension of the theory $T_0 = \{(\exists y_1)(\exists y_2)(\forall x)(x = y_1 \vee x = y_2)\}$ in language $L' = \langle \rangle$ with equality? (1 point)
 - (b) Are theories T and T_0 complete? Give an explanation. (1 point)